Assignment 9

Coverage: 16.4 in Text.

Exercises: 16.3 no 32, 33, 38; 16.4 no 7, 11, 14, 23, 26, 28, 35, 37, 39.

Hand in 16.4 no 14, 28, 35, 39 and Suppl. Problem no. 2 by Nov. 15.

Supplementary Problems

1. Verify Green's theorem when the region D is the rectangle $[0, a] \times [0, b]$.

2. Let D be the parallelogram formed by the lines $x+y = 1, x+y = 3, y = 2x-3, y = 2x+2$. Evaluate the line integral
$$\oint_C dx + 3xy\, dy$$
where C is the boundary of D oriented in anticlockwise direction. Suggestion: Try Green's theorem and then apply change of variables formula.

3. Find a potential for the vector field
$$\frac{-y}{x^2 + y^2}\mathbf{i} + \frac{x}{x^2 + y^2}\mathbf{j}\ ,$$
in the region obtained by deleting the line $(x, 0), x \le 0$, from $\mathbb{R}^2$.

4. Let $F = M\mathbf{i} + N\mathbf{j}$ be a smooth vector field which is defined in $\mathbb{R}^2$ except at the origin. Suppose that it satisfies the component test $M_y = N_x$. Show that for any simple closed curve γ enclosing the origin and oriented in positive direction, one has
$$\oint_\gamma M dx + N dy = \varepsilon \int_0^{2\pi} [-M(\varepsilon \cos\theta, \varepsilon \sin\theta) \sin\theta + N(\varepsilon \cos\theta, \varepsilon \sin\theta) \cos\theta]\, d\theta\ ,$$
for all sufficiently small ε. What happens when γ does not enclose the origin?